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$\mathcal{N} = 2$ solutions of massive type IIA and their Chern-Simons duals

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Abstract: We find explicit AdS_4 solutions of massive type IIA with $\mathcal{N} = 2$ supersymmetry obtained deforming with a Roman mass the type IIA supersymmetric reduction of the M theory background $AdS_4 \times M^{111}$. The family of solutions have $\mathrm{SU}(3) \times \mathrm{SU}(3)$ structure and isometry $\mathrm{SU}(3) \times \mathrm{U}(1)^2$. They are conjectured to be dual to three-dimensional $\mathcal{N} = 2$ Chern-Simons theories with generic Chern-Simons couplings and gauge group ranks.

Keywords: Flux compactifications, AdS-CFT Correspondence

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Contents

1 Introduction

Recent results on 2+1 dimensional superconformal Chern-Simons theories [1] have shed new light on the AdS_4/CFT_3 correspondence. A long standing problem in establishing this latter is the identification of the 2+1 dimensional superconformal gauge theories dual to AdS_4 supersymmetric backgrounds. In the past, attempts to find duals have focused on Yang-Mills theories flowing in the IR to superconformal fixed points [2–5]. It seems now that supersymmetric Chern-Simons theories can do a better job. The $\mathcal{N} = 6$ ABJM [1] model nicely incorporates all relevant features of a dual theory for the M theory background $AdS_4 \times S^7/\mathbb{Z}_k$, including the maximally supersymmetric $\mathcal{N} = 8$ case. Interestingly, for large k a better supergravity description is provided by the type IIA background $AdS_4 \times \mathbb{P}^3$. A similar construction has been extended to models with less supersymmetry. Examples of superconformal Chern-Simons theories with $\mathcal{N} = 3, 4, 5$ supersymmetry have been studied in [6–14]. The properties of $\mathcal{N} = 2$ theories have been investigated in [15, 16] and many models have been constructed and studied in [17–22]. For models with $\mathcal{N} = 2$ supersymmetry the reduction to type IIA is still less studied.

In this paper we consider the particular case of the $\mathcal{N} = 2$, M theory solution $AdS_4 \times M^{111}$, its reduction to type IIA and its supersymmetric deformations. In particular we find a family of $\mathcal{N} = 2$ supersymmetric AdS_4 vacua in massive type IIA supergravity with $\mathrm{SU}(3) \times \mathrm{U}(1)^2$ isometry, which include the $AdS_4 \times M^{111}$ reduction as special case.

The interest in such solutions is two-fold. On one side, they provide non-trivial examples of AdS_4 supersymmetric vacua of massive type IIA. In spite of the many known AdS_4

vacua, the picture we have so far is not exhaustive. In particular, most of the solutions have SU(3) structure [23–27], which is the simplest case but not the generic one. Only very recently, the conditions for $\mathcal{N} = 1$ supersymmetric solutions with generic $\mathrm{SU}(3) \times \mathrm{SU}(3)$ structure have been explicitly written and a type IIA solution was given [28]. As we will see later, in order to have non zero Roman mass and a running dilaton, type IIA backgrounds must have $\mathrm{SU}(3)\times\mathrm{SU}(3)$ structure. In that respect, our solution is one of the first non-trivial examples of AdS_4 backgrounds with $\mathrm{SU}(3) \times \mathrm{SU}(3)$ structure.

On the other side, the solution is also relevant for the $AdS_4 \times CFT_3$ correspondence. As noticed in [29, 30], the Roman mass can be interpreted as the overall Chern-Simons coupling in the dual gauge theory. More generally, all the integer Chern-Simons couplings and the ranks of the gauge groups should appear in the dual supergravity description. The authors of [29, 30] analysed the ABJM and ABJ models, finding solutions with $\mathcal{N} = 0$ and $\mathcal{N} = 1$ supersymmetry which are deformations of $AdS_4 \times \mathbb{P}^3$ and have a field theoretical interpretation. They also find analogous solutions with $\mathcal{N} = 2$ and $\mathcal{N} = 3$ supersymmetry at first order in perturbation theory [30], the entire solution still remaining to be found. The same argument applies to all $\mathcal{N} = 2$ Chern-Simons quivers. In particular it should always exist a massive type IIA deformation of the original supergravity solution which corresponds to a quiver with arbitrary Chern-Simons couplings and ranks. The deformation should preserve the same $\mathcal{N} = 2$ supersymmetry and the same global symmetry as the original theory. The solution we find in this paper corresponds to the supergravity backgrounds $M^{111}/\mathbb{Z}_k$. A candidate dual Chern-Simons quiver has been proposed in [15] and further studied in [17]. It is based on a superpotential with manifest $\mathrm{SU}(3) \times \mathrm{U}(1)^2$ symmetry. The existence of a supergravity solution with the same symmetry can be seen as a partial check of the correctness of the proposal.

We chose M^{111} because of its large global symmetry.[1] The isometries will allow to reduce the supersymmetry conditions to a set of ordinary first order equations. Fortunately these equations are not over-constrained and reduce to a pair of equations for two unknowns, which can be used to show the existence of a regular deformation. We will study the equations numerically and perturbatively. The $AdS_4 \times CFT_3$ correspondence suggests the existence of infinitely many other $\mathcal{N} = 2$ supergravity solutions associated with all Sasaki-Einstein manifolds with dual Chern-Simons quivers.[2] The methods of [29] and this paper still apply. However the smaller symmetry makes it more difficult to find explicit solutions to all orders. For example, in the case of another famous coset manifold Q^{111}, studied in the Chern-Simons context in [18, 21], the global symmetry $\mathrm{SU}(2)^3$ is reduced to a single SU(2). Generic Sasaki-Einstein manifolds are even more problematic having only abelian isometries.

The paper is organized as follows. In section 2 we review the M theory compactification on M^{111}, its reduction to type IIA and the proposed dual quiver. In section 3, we study

[1] In this case the global symmetry is larger than in the corresponding $\mathcal{N} = 2$ deformation of ABJM, where the full solution is still lacking. To the best of our knowledge, the type IIA reduction of M^{111} enjoys the largest global symmetry among the $\mathcal{N} = 2$ models with known or proposed Chern-Simons duals.

[2] For example, M^{111} belongs to the family of $Y^{p,q}(\mathbb{P}^2)$ Sasaki-Einstein manifolds [31, 32]. These models possess the same SU(3) symmetry and simply correspond to different choices of Chern-Simons couplings in the dual quiver. We shall discuss the relation of our results to $Y^{p,q}(\mathbb{P}^2)$ in the following.

how supersymmetry is realized in type IIA. In section 4, we study the conditions for supersymmetric massive type IIA deformations with $\mathrm{SU}(3)\times\mathrm{SU}(3)$ structure. We adopt an $\mathrm{SU}(3)\times\mathrm{U}(1)^2$ invariant ansatz, and we show that the system of supersymmetry equations is not over-constrained. We determine algebraically all the quantities appearing in the ansatz in terms of two unknowns for which we write a pair of coupled first order differential equations. In section 5, we analyze numerically and perturbatively the solution, showing that it is regular, and we determine the quantization conditions on the parameters. We then interpret the result in terms of the dual Chern-Simons quiver. In the two appendices, the conventions for the complex geometry of $\mathbb{P}^2$ and the supersymmetry conditions for $\mathrm{SU}(3)\times\mathrm{SU}(3)$ structures are reported.

2 The $AdS_4\times M^{111}$ background and its dual

M theory admits $\mathcal{N}=2$ supersymmetric Freund-Rubin solutions of the form $AdS_4\times H$ for every Sasaki-Einstein seven-manifold H. In this paper we will focus on the homogeneous space $H=M^{111}$, popular in the eighties, at the time of the Kaluza-Klein program, due to its intriguing isometry group, $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$. We will make use of this large isometry to find new AdS_4 solutions with $\mathcal{N}=2$ supersymmetry.

M^{111} is a U(1) bundle over $\mathbb{P}^2\times\mathbb{P}^1$. The metric reads [33, 34][3]

$$\mathrm{d}s^2_{M^{111}}=\left[\mathrm{d}s^2_{\mathbb{P}^2}+\frac{1}{8}(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2)+\frac{1}{64}(\mathrm{d}\tau+\lambda+2\cos\theta\mathrm{d}\phi)^2\right]. \tag{2.1}$$

τ is an angle with period 4π, while the one-form

$$\lambda=-3\sin^2\mu\,(\mathrm{d}\psi+\cos\tilde{\theta}\,\mathrm{d}\tilde{\phi}) \tag{2.2}$$

satisfies $\mathrm{d}\lambda=16j_0$, where j_0 is the Kähler form on $\mathbb{P}^2$. For convenience of the reader, the metric for $\mathbb{P}^2$ and its natural complex structure are reported in appendix A, together with a discussion of our conventions. M^{111} can be also described as the homogeneous space

$$\frac{\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)}{\mathrm{U}(1)\times\mathrm{U}(1)}\,. \tag{2.3}$$

Such a characterization helped in the study of the KK spectrum.

Various properties of M^{111}, relevant for the $AdS_4\times CFT_3$ correspondence, have been analysed a long time ago in [2], where the KK spectrum and the dimension of baryonic operators were studied. In the same paper, a candidate dual three-dimensional Yang-Mills theory was proposed. There are various indications now-days that a better candidate for the dual field theory is a Chern-Simons theory. An $\mathcal{N}=2$ Chern-Simons theory with the right moduli space was identified in [15] and further studied in [17]. The theory is based on a quiver with three gauge groups and three sets of chiral fields U_i,V_i,W_i, with $i=1,2,3$,

[3] In this and the following section, we set for simplicity the cosmological constant $\Lambda=-3|\mu|^2=12$. We will set also $g_s=1$ for the asymptotic value of the dilaton in type IIA. These quantities can be easily reintroduced by rescaling the metric (2.1) and the RR fluxes $F_{RR}\to\frac{1}{g_s}F_{RR}$.

transforming in the $(N, \bar{N}, 0), (0, N, \bar{N})$ and $(\bar{N}, 0, N)$ representation of the gauge groups, respectively. They interact with the superpotential

$$\mathcal{W} = \epsilon_{ijk} U_i V_j W_k \,. \tag{2.4}$$

There is a Chern-Simons coupling k_i for each gauge group but no Yang-Mills terms. The theory has a global SU(3) symmetry rotating the indices $i = 1, 2, 3$ of U, V, W. Note that this theory has the same field content and superpotential as the quiver associated to D3-branes sitting at a $\mathbb{C}^3/\mathbb{Z}_3$ singularity and describing a $3+1$ $\mathcal{N} = 1$ superconformal theory.

With the choice of Chern-Simons couplings $k_1 = k, k_2 = k, k_3 = -2k$, the moduli space of the theory is the Calabi-Yau cone $C(M^{111})/\mathbb{Z}_k$, and the theory describes the M theory background $AdS_4 \times M^{111}/\mathbb{Z}_k$. It is interesting to see in details how this happens [15, 17]. As discussed in [11, 13, 15], the D-term equations in a $\mathcal{N} = 2$ Chern-Simons theory are modified by a term proportional to the Chern-Simons coupling

$$D_a = k_a \sigma \,. \tag{2.5}$$

In this formula, D_a is the momentum map for the action of the a-th U(1) gauge field on the elementary fields, and σ is an auxiliary field in the gauge multiplets. More explicitly, the previous equations read

$$\begin{aligned} \sum_{i=1}^{3} |U_i|^2 - |V_i|^2 &= k_1 \sigma \,, \\ \sum_{i=1}^{3} |V_i|^2 - |W_i|^2 &= k_2 \sigma \,, \\ \sum_{i=1}^{3} |W_i|^2 - |U_i|^2 &= k_3 \sigma \,. \end{aligned} \tag{2.6}$$

These equations should be supplemented by the F-term conditions

$$U_i V_j = U_j V_i \qquad V_i W_j = V_j W_i \qquad W_i U_j = W_j U_i \qquad\qquad i \neq j \,. \tag{2.7}$$

The sum of the three equations in (2.6) is zero, reflecting the fact that the overall U(1) acts trivially on the fields. The difference of the first two equations in (2.6) imposes the vanishing of the momentum map for $\mathrm{U}(1)_1 - \mathrm{U}(1)_2$. The last equation just determines the value of σ. We see that effectively we should only divide by the (complexified) gauge group $\mathrm{U}(1)_1 - \mathrm{U}(1)_2$[4] which acts with charge $+2$ on U_i and charge -1 on V_i and W_i. $\mathrm{U}(1)_3$ is broken to $\mathbb{Z}_k$ by the Chern-Simons interactions and remains as a global symmetry.

For $k = 1$ the theory has symmetry $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$, where SU(3) acts on the i, j, k indices, U(1) is the R-symmetry, which rotates all the fields (U, V, W) with the

[4] As usual, a momentum map condition and the modding by the corresponding gauge group can be combined into the modding by the complexified gauge group.

JHEP09(2009)107

same charge, and, finally, SU(2) is the enhancement of $\mathrm{U}(1)_3$ obtained by considering the doublets $R_i^A = (V_i, W_i)$. The gauge invariant chiral operators are given by

$$O_n = (URR)^n \,, \tag{2.8}$$

where the SU(3) and SU(2) indices are symmetrized due to the F-term conditions. Since the superpotential $\mathcal{W}$ must have R-charge two, O_n has R-charge $2n$. In conclusion, there is exactly one chiral multiplet with R-charge $2n$ transforming in the $[3n, 0]$ representation of SU(3) and the $2n$ representation of SU(2). We recognize the KK spectrum of M theory compactified on M^{111} [2]. A more geometrical proof, based on toric geometry, that the moduli space of the Chern-Simons theory is $C(M^{111})$ can be found in [15, 17, 35].

For $k \neq 1$ we have to mod by $\mathbb{Z}_k \in \mathrm{U}(1)_3$, which acts with charge $+1$ on W_k and charge -1 on V_j. The moduli space is now $C(M^{111})/\mathbb{Z}_k$ and the SU(2) symmetry is broken to U(1). $\mathbb{Z}_k$ acts indeed on a circle in M^{111} reducing its radius. For large k the compactification on $AdS_4 \times M^{111}/\mathbb{Z}_k$ is effectively reduced to a type IIA compactification, as in the ABJM model. It is easy to identify the action of $\mathbb{Z}_k$ with a shift of ϕ in the metric (2.1). Reducing along ϕ gives a supersymmetric type IIA background with non trivial dilaton, F_2 flux and $\mathrm{SU}(3) \times \mathrm{U}(1)^2$ symmetry. The metric is

$$\mathrm{d}s_{10}^2 = e^{2A}\, \mathrm{d}s_{AdS_4}^2 + \mathrm{d}s_6^2 \,, \tag{2.9}$$

where the metric of the six-dimensional compact manifold is

$$\mathrm{d}s_6^2 = e^{2A}[\mathrm{d}s_{\mathbb{P}^2}^2 + \frac{1}{8}\,\mathrm{d}\theta^2 + \frac{1}{32}\,\frac{\sin^2\theta}{1+\sin^2\theta}(\mathrm{d}\tau + \lambda)^2] \,. \tag{2.10}$$

The term $\mathrm{d}\tau + \lambda$ in the original metric determines a non trivial RR two-form

$$F_2 = \mathrm{d}\left[\frac{\cos\theta}{2(1+\sin^2\theta)}(\mathrm{d}\tau + \lambda)\right] \,. \tag{2.11}$$

Finally, the warp factor is proportional to the dilaton and is given by

$$e^{2A} = e^{2\varphi/3} = \frac{1}{4}\sqrt{1+\sin^2\theta} \,. \tag{2.12}$$

Type IIA reductions of M^{111} and other Sasaki-Einstein manifolds have been considered in the past [36, 37]. The reduction was performed on the obvious U(1) circle bundle with the result of breaking supersymmetry. The natural U(1) bundle of the $\mathbb{P}^2 \times \mathbb{P}^1$ fibration corresponds indeed to the R-symmetry. A reduction along the $\mathbb{Z}_k$ action, on the other hand, preserves supersymmetry since $\mathbb{Z}_k$ is a subgroup of the global SU(2) symmetry.

In the following section we will verify that the previous solution is $\mathcal{N} = 2$ supersymmetric. We will then find an $\mathcal{N} = 2$ supersymmetric deformation of this solution in massive type IIA preserving SU(3) symmetry. As suggested in [29] such solutions should correspond to the case $\sum_a k_a \neq 0$ and possibly generic N_i. The existence of a solution is predicted by the Chern-Simons theory since a modification of $\sum_a k_a$ and N_i does not affect the superpotential and preserves the SU(3) symmetry. It is quite remarkable that such supergravity solution actually exists.

3 Undeformed solution

We now verify that the background (2.9)–(2.12) is a solution of type IIA supergravity with $\mathcal{N} = 2$ supersymmetry. M^{111} admits two real Killing spinors which give, after reducing to six dimensions, one Weyl spinor each and, hence, $\mathcal{N} = 2$ supersymmetry. We use the language of Generalised Complex Geometry [38, 39] which is briefly reviewed in appendix B.

Since each Killing spinor can be seen as defining an SU(3) structure, a convenient way to check supersymmetry is to look for two pairs of SU(3) structure pure spinors satisfying [40, 41]

$$(\mathrm{d} - H\wedge)(e^{3A-\varphi}\,\mathrm{Im}\,\Phi_-) = -3e^{2A-\varphi}\mu\,\mathrm{Im}\,\Phi_+ + \frac{e^{4A}}{8} * \lambda(F)\,, \tag{3.1}$$

$$(\mathrm{d} - H\wedge)(e^{2A-\varphi}\Phi_+) = -2\mu e^{A-\varphi}\,\mathrm{Re}\,\Phi_-\,, \tag{3.2}$$

where μ is related to the cosmological constant in AdS_4 by $\Lambda = -3|\mu|^2$. By changing phases in the spinors we can always take μ real, and we will do so in the following.

We can write the SU(3) structure pure spinors as in (B.9), choosing the parametrization $a = ie^{A/2}e^{i(\rho+\alpha)}$ and $x = e^{A/2}e^{i\alpha}$

$$\Phi_+ = \frac{i}{8}e^{i\rho}e^{A}\,e^{-iJ}\,, \qquad\qquad \Phi_- = -\frac{i}{8}e^{i(\rho+2\alpha)}e^{A}\,\Omega\,. \tag{3.3}$$

The fibered structure of the six-dimensional metric suggests a natural splitting into base and fiber directions for the choice of the holomorphic three-form Ω and Kähler form J

$$\Omega = i\omega\wedge z\,, \qquad\qquad J = j + \frac{i}{2}z\wedge\bar{z}\,. \tag{3.4}$$

Here z is a one form on the S^2 fiber

$$z = -\frac{ie^{A}}{2\sqrt{2}}\left[\mathrm{d}\theta + i\frac{\sin\theta}{2\sqrt{1+\sin^2\theta}}(\mathrm{d}\tau + \lambda)\right]\,, \tag{3.5}$$

while j and ω are a rotation of the natural complex structure on $\mathbb{P}^2$ (see appendix A for notations)

$$\begin{aligned} j &= e^{2A}(\cos\gamma\,j_0 + \sin\gamma\,\mathrm{Re}\,\hat{\omega}_0)\,, \\ \omega &= e^{2A}[(-\sin\gamma\,j_0 + \cos\gamma\,\mathrm{Re}\,\hat{\omega}_0) + i\,\mathrm{Im}\,\hat{\omega}_0]\,, \end{aligned} \tag{3.6}$$

with γ a function of the angle θ on the two-sphere.

With the choice (3.3), the equation (3.2) for the even pure spinor reduces to the two conditions

$$\mathrm{d}(3A - \varphi + i\rho) = 0\,, \tag{3.7}$$

$$\mathrm{d}J - iH = -2\mu e^{-i\rho}e^{-A}\,\mathrm{Re}(-ie^{i(\rho+2\alpha)}\,\Omega)\,. \tag{3.8}$$

The first equation sets ρ to a constant and implies the same proportionality as in (2.12) between the dilaton and the warp factor

$$\varphi = 3A\,. \tag{3.9}$$

JHEP09(2009)107

Since $H = 0$ in the solution, from the second equation we see that $e^{i\rho}$ must be real. Choosing $\rho = 0$, it is straightforward to verify that (3.8) is satisfied by the ansatz (3.4)–(3.6) with $\alpha = \pi/4$, $\mu = -2$ and

$$\cos\gamma = \frac{\cos\theta}{\sqrt{1+\sin^2\theta}}\,, \qquad \sin\gamma = \sqrt{2}\frac{\sin\theta}{\sqrt{1+\sin^2\theta}}\,. \tag{3.10}$$

Similarly, equation (3.1) for Φ_- gives the closure of the imaginary part of Ω

$$\mathrm{d}[e^{-A}\,\mathrm{Im}(\Omega)] = 0\,, \tag{3.11}$$

and the RR fluxes

$$F_4 = 0\,, \qquad e^{4A} * F_6 = -6\,, \tag{3.12}$$

$$F_0 = 0\,, \qquad e^{4A} * F_2 = -\mathrm{d}(e^A\,\mathrm{Re}\,\Omega) + 3J^2\,. \tag{3.13}$$

Again, it is easy to check that the ansatz (3.4)–(3.6) solves the equation for the closure of $\mathrm{Im}\,\Omega$ and that the F_2 defined in (2.11) satisfies (3.13). Finally, we can take the equation for F_6 as a definition of the cosmological constant in the solution.

The discussion above proves that, reducing the M^{111} background, we obtain a solution of IIA with one supersymmetry. We still have to look for the second supersymmetry. However, it is immediate to construct a second pair of pure spinors satisfying the equations (3.1),(3.2). These have the same form as in (3.3), with Ω and J defined as in (3.4), but with a different complex structure obtained by a change of sign in the coordinates on the base

$$j = e^{2A}(-\cos\gamma\, j_0 + \sin\gamma\,\mathrm{Re}\,\hat{\omega}_0)\,, \tag{3.14}$$

$$\omega = e^{2A}[(\sin\gamma\, j_0 + \cos\gamma\,\mathrm{Re}\,\hat{\omega}_0) - i\,\mathrm{Im}\,\hat{\omega}_0]\,, \tag{3.15}$$

$$z = -\frac{ie^A}{2\sqrt{2}}\left[\mathrm{d}\theta - i\frac{\sin\theta}{2\sqrt{1+\sin^2\theta}}(\mathrm{d}\tau+\lambda)\right]. \tag{3.16}$$

As already mentioned, there are other solutions of type IIA with $\mathcal{N} = 2$ supersymmetry and $SU(3)$ structure with the same global symmetry. M^{111} belongs indeed to the larger family of Sasaki-Einstein manifolds $Y^{p,q}(\mathbb{P}^2)$ with $\mathrm{SU}(3)\times\mathrm{U}(1)^2$ isometry. These solutions correspond to different choices of Chern-Simons couplings with $p = k_1 + k_2, k = 2k_1 + k_2$ for $k_1, k_2 \geq 0$ [32]. The reduction to a type IIA background can be found and studied similarly.

4 Deformed solution

Now we come to massive type IIA deformations of the previous background that still preserve $\mathcal{N} = 2$ supersymmetry and have SU(3) global symmetry. These are obtained by introducing the Roman mass F_0.

We choose an SU(3) invariant ansatz for the metric

$$\mathrm{d}s^2_{10} = e^{2A(\theta)}\mathrm{d}s^2_{AdS_4} + \mathrm{d}s^2_6\,, \tag{4.1}$$

where the six-dimensional compact manifold is still a 2-dimensional fibration over $\mathbb{P}^2$

$$ds_6^2 = e^{2B(\theta)}\left[ds^2_{\mathbb{P}^2} + \frac{1}{8}\,\epsilon^2(\theta)d\theta^2 + \frac{1}{64}\Gamma^2(\theta)(d\tau + \lambda)^2\right]. \tag{4.2}$$

It is still convenient to define a one-form on the S^2 fiber

$$z = -ie^{B(\theta)}e^{-i\nu(\theta)}\left[\frac{\epsilon(\theta)}{2\sqrt{2}}d\theta + \frac{i}{8}\Gamma(\theta)(d\tau + \lambda)\right]. \tag{4.3}$$

The phase ν will be fixed shortly. Since we can redefine θ, one of the functions in the ansatz is redundant. We will use the freedom to change coordinate later.

For the fluxes, we take the natural SU(3) invariant ansatz

$$\begin{aligned} F_0 &= f_0\,,\\ F_2 &= f_2(\theta)\,j_0 + \frac{i}{2}\,g_2(\theta)\,z\wedge\bar z\,,\\ F_4 &= f_4(\theta)\,j_0\wedge j_0 + \frac{i}{2}\,g_4(\theta)\,z\wedge\bar z\wedge j_0\,,\\ F_6 &= e^{4B(\theta)}\frac{i}{4}\,f_6(\theta)\,z\wedge\bar z\wedge j_0^2\,,\\ H &= h(\theta) j_0\wedge d\theta\,. \end{aligned} \tag{4.4}$$

The ansatz is SU(3) invariant since j_0 and λ are.

It is easy to check that, when $F_0 \neq 0$, the supersymmetry equations for SU(3) structure pure spinors require constant dilaton. Since the dilaton is running even in the unperturbed solution, we are led to consider solutions with SU(3) $\times$ SU(3) structure. We can write the SU(3) $\times$ SU(3) structure pure spinors as in (B.10), choosing the dielectric ansatz [30, 42]

$$\begin{aligned} a &= i\cos\phi\, e^{i\rho}e^{i\alpha}e^{A/2}\,, & x &= \cos\phi\, e^{i\alpha}e^{A/2}\,,\\ b &= -i\sin\phi\, e^{i\rho}e^{-i\alpha}e^{A/2}\,, & y &= \sin\phi\, e^{-i\alpha}e^{A/2}\,. \end{aligned} \tag{4.5}$$

Here ρ, ϕ and α are functions of the angle θ on the fiber.

An SU(3) $\times$ SU(3) structure corresponds to a $4+2$ splitting on the internal metric, determined by a vector z. It is natural for us to use the splitting into $\mathbb{P}^2$ and S^2: z has been defined above and j,ω are defined as in (3.6), with a possibly different function γ. This is the natural generalization of what we used for the undeformed solution. A quick analysis shows that we can consistently choose $\alpha = \pi/4$ as in the undeformed case.

We need to solve (3.1), (3.2) for the new pure spinors. To simplify our notations, let us notice that the 10-dimensional metric is invariant under the simultaneous rescaling of μ and e^A, so that we can reabsorb μ in the definition of A.[5] We use this freedom to fix again $\mu = -2$. The one-form component of (3.2),

$$d(ie^{3A-\varphi}\, e^{i\rho}\, \cos 2\phi) = -4e^{2A-\varphi}\,\mathrm{Re}(ie^{i\rho}\, \sin 2\phi\, z)\,, \tag{4.6}$$

[5] Since $\Phi \sim e^A$, the pure spinor equations can be reformulated in terms of e^A/μ and $e^{3A-\varphi}$ and the phases in the dielectric ansatz. The arbitrary constant in $e^{3A-\varphi}$ can be reabsorbed by a rescaling of the RR fluxes and will be set to one in the text.

immediately gives a lot of information. The fact that the right-hand side must be closed implies that it is proportional to $\mathrm{d}\theta$. This can be obtained by choosing $\nu = \rho$ in (4.3). The imaginary part of the previous equation then fixes the dilaton

$$e^{3A-\varphi} = \frac{1}{\cos 2\phi \cos \rho} \,. \tag{4.7}$$

Again, for simplicity, we omitted an arbitrary constant. As easily seen from the equations, the constant can be reintroduced by rescaling the RR fluxes, $F \to F/g_s$.

The remaining equations give several differential and algebraic constraints for few unknown functions, $A, B, \Gamma, \epsilon, \rho, \phi, \gamma, f_i, \gamma_i$. To these constraints we have to add the Bianchi identities for fluxes

$$\mathrm{d}F - H \wedge F = 0 \,. \tag{4.8}$$

There are clearly more equations than unknowns. However, this formidable system is not over-constrained and, with some patience, it can be reduced to a pair of linear differential equations for two unknowns. All other quantities can be obtained algebraically and the Bianchi identities are automatically satisfied.

In order to simplify the resulting set of equations, it is convenient to use the freedom of redefining θ. We can always choose coordinates where $\gamma(\theta)$ is the function defined in (3.10). With this choice we can write a pair of linear differential equations for the quantities ϕ and $w = 4e^{2(B-A)}$,

$$\begin{aligned} \phi' &= \frac{\sin 4\phi \, \cot\theta \, [w - 2(\sin^2 2\phi + 2\tan^2\theta)]}{4[w\,(1+\sin^2\theta) - 2\cos^2 2\phi\,(2\sin^2\theta + \cos^2\theta \, \sin^2 2\phi)]} \,, \\ w' &= \frac{-w \, \cot\theta\,(\sin^2 2\phi + 2\tan^2\theta)(w - 4 - 4\sin^2 2\phi)}{2[w\,(1+\sin^2\theta) - 2\cos^2 2\phi\,(2\sin^2\theta + \cos^2\theta \, \sin^2 2\phi)]} \,. \end{aligned} \tag{4.9}$$

All other quantities are then determined in terms of the previous ones. It turns out that $\tan\rho = -\cot\theta \sin 2\phi/\sqrt{2}$. The functions appearing in the metric are given by

$$\begin{aligned} e^{4A} &= \frac{2\sqrt{2}}{f_0} \frac{\sin 2\phi\,(1+\sin^2\theta)}{\cos^2 2\phi \, \sin 2\theta} \,, \\ \epsilon &= 2\sqrt{w} \frac{\sqrt{4\tan^2\theta + 2\sin^2 2\phi}}{\sin 2\phi\,(w - 4\tan^2\theta - 2\sin^2 2\phi)} \, \phi' \,, \\ \Gamma &= \frac{2}{\sqrt{w}} \, \frac{\cos\theta \, \sqrt{2\tan^2\theta + \sin^2 2\phi}}{\sqrt{1+\sin^2\theta}} \,. \end{aligned} \tag{4.10}$$

JHEP09(2009)107

The fluxes read

$$
\begin{aligned}
h &= \sqrt{2}e^{2A}\, w\, \frac{\sin\theta\,(\sin^2 2\phi + 2\tan^2\theta)}{\sqrt{(1+\sin^2\theta)}\,[w-(2\sin^2 2\phi + 4\tan^2\theta)]}\,\phi'\,,\\
f_2 &= e^{-2A}\,\frac{[w(1+\sin^2\theta) - 4\cos^2\theta\,(2\tan^2\theta + \sin^2 2\phi)]}{2\sqrt{1+\sin^2\theta}\,\cos\theta\cos 2\phi}\,,\\
g_2 &= -2e^{-4A}\frac{[3w(1+\sin^2\theta) - 8\cos^2\theta\,(2\tan^2\theta + \sin^2 2\phi)]}{(1+\sin^2\theta)w}\,,\\
f_4 &= -\,\frac{w[w(1+\sin^2\theta) - 8\cos^2\theta\,(2\tan^2\theta + \sin^2 2\phi)]\sin 2\phi}{8\sqrt{2}\sin 2\theta\,\cos^2 2\phi}\,,\\
g_4 &= \frac{1}{2\sqrt{2}}e^{-2A}\,\frac{[3w(1+\sin^2\theta) - 4\cos^2\theta\,(2\tan^2\theta + \sin^2 2\phi)]\tan 2\phi}{\sin\theta\,\sqrt{1+\sin^2\theta}}\,,\\
f_6 &= \frac{3}{\sqrt{2}}f_0\frac{\sin 2\theta\cos^2 2\phi}{(1+\sin^2\theta)\sin 2\phi}.
\end{aligned}
\tag{4.11}
$$

In all the expressions above f_0 is set to a constant by the Bianchi identities.

The solution has $\mathcal{N}=2$ supersymmetry. The second supersymmetry is obtained, as in the unperturbed case, by changing complex structure as in (3.14)

$$
j = e^{2A}(-\cos\gamma(\theta)\,j_0 + \sin\gamma(\theta)\,\mathrm{Re}\,\omega_0)\,, \tag{4.12}
$$

$$
\omega = e^{2A}[(\sin\gamma(\theta)\,j_0 + \cos\gamma(\theta)\,\mathrm{Re}\,\omega_0) - i\,\mathrm{Im}\,\omega_0]\,, \tag{4.13}
$$

$$
z = -ie^{B(\theta)}e^{-i\nu(\theta)}\left[\frac{\epsilon(\theta)}{2\sqrt{2}}\mathrm{d}\theta - \frac{i}{8}\Gamma(\theta)(\mathrm{d}\tau+\lambda)\right]. \tag{4.14}
$$

The pure spinors are given by the dielectric ansatz with $\phi\to-\phi$, $\rho\to\rho$. The supersymmetry equations are then satisfied with the same metric and fluxes as before. The relations (4.9)–(4.11) remain true.

5 Analysis and interpretation of the solution

We were not able to solve analytically the system of equations (4.9)–(4.11), but we can study the properties of the solution using perturbation theory and numerical analysis.

We have solved the equations (4.9)–(4.11) up to third order. The idea is to define a perturbative expansion where ϕ, ρ and the fluxes H, F_4, F_0 receive corrections at odd orders, while the metric and fluxes F_2, F_6 receive corrections at even orders. The constant f_0 has an odd expansion, while μ an even one. At first order we find

$$
\phi^{(I)} = 2c^{(I)}\cos 2\theta\,, \tag{5.1}
$$

with fluxes

$$
\begin{aligned}
h^{(I)} &= \sqrt{2}c^{(I)}\sin^3\theta\,,\\
f_4^{(I)} &= 32\sqrt{2}\,c^{(I)}\,\frac{3\cos 2\theta - 1}{\cos 2\theta - 3}\,,\\
g_4^{(I)} &= -64\,c^{(I)}\,\frac{\cos 3\theta - 13\cos\theta}{(3-\cos 2\theta)^{3/2}}\,.
\end{aligned}
\tag{5.2}
$$

The constant f_0 requires a word of caution. As one can see from equation (4.10), the limit $f_0 \to 0$ is singular. The correct unperturbed limit is obtained by sending simultaneously $f_0 \to 0$ and $\phi \to 0$. The solution of section 3 is obtained by setting

$$f_0^{(I)} = 64\sqrt{2}c^{(I)} . \tag{5.3}$$

The metric receives corrections at second order, which can be easily determined and which depend on a second arbitrary constant. We do not report the expressions here. We have checked the regularity of the resulting metric for all θ, and, in particular, at the North and South poles and at the equator of the sphere. Up to third order, the expansion gives a perfectly regular solution of type IIA supergravity.

More generally, we can study the regularity of the metric near the North and South pole by expanding θ around 0 and π. By solving the equations (4.9)–(4.11) near $\theta = 0$ we find

$$\begin{aligned} \phi &= \phi_1\theta - \left(\frac{2\phi_1}{3} + \frac{4\phi_1^3}{3}\right)\theta^3 + O(\theta^5) , \\ w &= w_0 + \frac{1}{2}(4 - w_0)(1 + 2\phi_1^2)\,\theta^2 + O(\theta^4) . \end{aligned} \tag{5.4}$$

An identical expression holds at the South pole with different parameters $\tilde{\phi}_1, \tilde{w}_0$. The S^2 metric will be smooth near the poles if it reduces to the flat metric in polar coordinates. It is easy to check, using equations (4.10), that

$$\epsilon \to 2\sqrt{\frac{1+2\phi_1^2}{w_0}} , \qquad\qquad \Gamma \to \sqrt{2}\epsilon(0)\theta , \tag{5.5}$$

at the North pole, and similarly, with $\phi_1 \to \tilde{\phi}_1, w_0 \to \tilde{w}_0$, at the South pole. The vector z has then an expansion (fixing an arbitrary point in $\mathbb{P}^2$)

$$z \sim \mathrm{d}(\theta - \theta_P) + i(\theta - \theta_P)\frac{\mathrm{d}\tau}{2} \tag{5.6}$$

at both poles. Since τ has period 4π, this guaranties the regularity of the metric. Warp factors and fluxes are similarly computed and are regular. The expansion of the equations near the equator is also smooth.

We can further study the solution by numerical analysis. We can use ϕ_1, w_0 as parameters labeling the solutions of the system (4.10). The analysis shows that there is a two-parameter family of regular solutions departing from the unperturbed one. The shape of ϕ and e^{2A} is shown in figure 1 for special values of the parameters.

The solution depends on four arbitrary parameters: the asymptotic value of the dilaton, g_s, and the radii of AdS_4, $\mathbb{P}^2$ and S^2 (these are functions of f_0, ϕ_1, w_0). To these, we can add the zero modes of the B-field on the two two-cycles in the solution,[6] for a total of six parameters. Their values are constrained by flux quantization. f_0 is interpreted as the period of a zero form and must be an integer. For the other RR forms, we define Page

[6]We thank Alessandro Tomasiello for an enlightening discussion on this point.

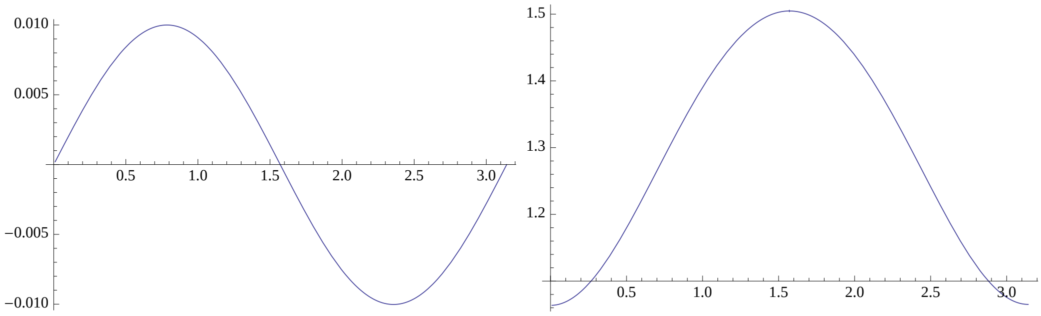

Figure 1. Graphics of ϕ and e^{2A} for the values of parameters $\phi_1 = 0.02$ and $w_0 = 4$. The symmetry $\theta \to \pi - \theta$ is only present for the value $w_0 = 4$.

charges associated to the quantities $\tilde{F}_2 = F_2 - Bf_0$, $\tilde{F}_4 = F_4 - B \wedge F_2$ and $\tilde{F}_6 = F_6 - B \wedge F_4$. As we already said, the Bianchi identities are satisfied and these forms are closed. It follows from the ansatz (4.4) that we can conveniently write them as

$$F_2 - Bf_0 = \mathrm{d}\left[f_2 \frac{(\mathrm{d}\tau + \lambda)}{16}\right], \qquad F_4 - B \wedge F_2 = \mathrm{d}\left[f_4 \frac{(\mathrm{d}\tau + \lambda)}{16}\right]. \tag{5.7}$$

We have to impose that the periods of these forms on all non trivial internal cycles are integer (in suitable units). A basis for the two-cycles is given by a copy of $\mathbb{P}^1 \in \mathbb{P}^2$ at $\theta = 0$ and by a copy of S^2 at a chosen point in $\mathbb{P}^2$. Similarly, a basis for the four-cycles is given by $\mathbb{P}^2$ and $\mathbb{P}^1 \times S^2$. Equation (5.7) allows to evaluate easily the periods. For example, without including the zero modes of the B field, we have

$$\int_{S^2} \tilde{F}_2 = [f_2(\pi) - f_2(0)]\frac{\pi}{4}, \qquad \int_{\mathbb{P}^1} \tilde{F}_2 = f_2(0) \int_{\mathbb{P}^1} j_0 \,. \tag{5.8}$$

All these integrals can be expressed in terms of the parameters of the solutions. The the zero modes of B can be easily included. We obtain six quantization conditions for the six parameters in the solution.

Now we can compare the results with the field theory expectations. The original Chern-Simons theory can be deformed preserving $\mathcal{N} = 2$ supersymmetry by changing the ranks N_i and by relaxing the condition $\sum_a k_a = 0$. Since the superpotential is unchanged, the SU(3) global symmetry is also preserved. The most general SU(3) invariant $\mathcal{N} = 2$ Chern-Simons quiver depends on six integer parameters, the three ranks N_i of the gauge groups and the three Chern-Simons couplings k_a. We have room to describe all these in our deformed solution. The precise identification of the supergravity parameters with the field theory ones is complicated. A very rough identification is as follows. The original parameters N and k are still described by the constants in the dilaton and the AdS radius. The $\sum_a k_a$ can be associated with f_0 [29], while the difference between the ranks of the gauge groups should correspond to the zero modes of the B-field [12]. The extra parameter corresponds to varying the ratio of the $\tilde{F}_2$ periods on the two-cycles. In the unperturbed solution, this would correspond to replacing M^{111} with a generic member of the family $Y^{p,q}(\mathbb{P}^2)$. It is reasonable that our solution for $f_0 \neq 0$ already describes the quiver with generic k_i, as the parameters of the supergravity solution suggest. In fact, one can explicitly verify that our set of equations in the limit $f_0 \to 0$ contain, in addition to the solution given in section 3 with $w = 4$, other solutions with non-trivial w corresponding to the dimensional reduction of the manifolds $Y^{p,q}(\mathbb{P}^2)$ for generic $(k_1, k_2, -k_1 - k_2)$ [31, 32]. The different values of k_a appear in type IIA as periods of $\tilde{F}_2$.

As a final check, we can insert D2-brane probes in the background [30]. As discussed in [30, 43, 44], the supersymmetry conditions for a probe D2 requires

$$\mathrm{d}(\mathrm{Re}\,\Phi|_{(0)}) \equiv \mathrm{d}(\tan\rho) = 0\,. \tag{5.9}$$

Using the perturbative and numerical expansion of the solution it is easy to check that a supersymmetric locus for D2 probes corresponds to $\theta = \pi/2$. On this locus, the τ fibration over $\mathbb{P}^2$ reproduces $S^5/\mathbb{Z}_3$. The moduli space for a supersymmetric D2 is obtained by adding the radial direction in AdS_4 giving the cone over $S^5/\mathbb{Z}_3$, which, as a complex three-dimensional manifold, is $\mathbb{C}^3/\mathbb{Z}_3$. This result matches the moduli space of Chern-Simons quivers for $\sum_a k_a \neq 0$, which becomes three-dimensional. This is simple to see from equations (2.6). The sum of the three equations gives $\sigma \sum_a k_a = 0$, which implies $\sigma = 0$. The equations (2.6) become standard D-term constraints for all gauge groups. In particular, we mod also by $\mathrm{U}(1)_3$, reducing the complex dimension of the moduli space of one unit. The moduli space is then the same as for the $3+1$ dimensional superconformal theory based on the same quiver, that is $\mathbb{C}^3/\mathbb{Z}_3$.[7]

6 Conclusions

In this paper we have considered an AdS_4 solution of massive type IIA supergravity with $\mathcal{N} = 2$ supersymmetry and $\mathrm{SU}(3) \times \mathrm{U}(1)^2$ global symmetry. Having non zero Roman mass and non trivial dilaton, this solution has $\mathrm{SU}(3) \times \mathrm{SU}(3)$ structure.

The $AdS_4 \times CFT_3$ correspondence actually suggests the existence of many supersymmetric AdS_4 vacua still to be found. The method in this paper can be applied to various Sasaki-Einstein metrics. The generalization to $Y^{p,q}(\mathbb{P}^2)$ is already contained in our deformed equations. More interesting would be to find solutions for the family $Y^{p,q}(\mathbb{P}^1 \times \mathbb{P}^1)$, which includes Q^{111}. In this case, the global symmetry is typically reduced to $\mathrm{SU}(2)^2$ and the corresponding solution can be harder to find. Even in the apparently simple case of ABJM, the solution at all order is still lacking. The perturbative method of [30] however always applies. It would be interesting to perform a perturbative expansion for the Sasaki-Einstein manifolds where a dual quiver has been proposed. This would be an interesting check of the correctness of the proposal, which is still unclear since many standard checks of the correspondence cannot be fully performed in 2+1 dimensions.

One can also study deformations with $\mathcal{N} = 1$ supersymmetry. Many solutions are still predicted by the correspondence, since the dual quiver can be typically deformed to $\mathcal{N} = 1$, usually loosing some global symmetry. Alternatively, one can forget about the dual

[7] Recall that the case $\sum_a k_a = 0$, corresponding to zero Roman mass f_0, is special: the moduli space of the quiver is of complex dimension four [13, 15, 16] and we can uplift the solution to M theory. For $\sum_a k_a \neq 0$, differences between models seem to disappear: as discussed in the text, the moduli space of all $Y^{p,q}(\mathbb{P}^2)$ quivers degenerates to $\mathbb{C}^3/\mathbb{Z}_3$ and this is well captured by the type IIA geometry we found.

interpretation, and search for $\mathcal{N}=1$ SU(3) $\times$ SU(3) solutions with large global symmetry. As shown in this paper, although apparently complicated, the supersymmetry conditions can be sometimes simplified and reduced to a simple set of equations. In particular, it would be interesting to see whether there exist other $\mathcal{N}=1$ solutions of the equations we wrote with SU(3) global symmetry. This is left for future work.

Note added. As noted in [45], $\mathbb{P}^2$ can be replaced by any compact Kähler-Einstein base $\mathcal{B}$ without affecting the conditions of supersymmetry and without modifying any of the formulae in this paper. In fact, all the equations follow from the existence of a trio of forms λ, j_0, ω_0 on the base satisfying $\mathrm{d}\lambda = 16 j_0$, $\mathrm{d}\omega_0 = i\frac{\lambda}{2}\wedge\omega_0$, which is a distinctive feature of all Kähler-Einstein manifolds. For any M theory background $Y^{p,q}(\mathcal{B})$, the equations in this paper define an AdS_4 massive type IIA deformation with non zero Roman mass and with the same global symmetries of $\mathcal{B}$. This agrees with the expectation that the corresponding quiver can be deformed by changing the Chern-Simons couplings and the ranks of the gauge groups without breaking the global symmetries. This construction applies in particular to $\mathcal{B}=\mathbb{P}^1\times\mathbb{P}^1$ which can be used to describe Q^{111} and its quotients.

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A Conventions for $\mathbb{P}^2$

In our conventions, the metric on $\mathbb{P}^2$ is normalized as

$$\mathrm{d}s^2_{\mathbb{P}^2} = \frac{3}{4}\left[\mathrm{d}\mu^2 + \frac{1}{4}\sin^2\mu\,\cos^2\mu^2(\mathrm{d}\psi+\cos\tilde\theta\mathrm{d}\tilde\phi)^2 + \frac{1}{4}\sin^2\mu(\mathrm{d}\tilde\theta^2+\sin^2\tilde\theta\mathrm{d}\tilde\phi^2)\right] \tag{A.1}$$

and $R^{\mathbb{P}^2}_{ab} = \frac{9}{2}g^{\mathbb{P}^2}_{ab}$. We chose vierbein,

$$\begin{aligned} e^1 &= \frac{\sqrt{3}}{4}\,r\sin\mu\cos\mu(\mathrm{d}\psi+\cos\tilde\theta\,\mathrm{d}\tilde\phi)\,,\\ e^2 &= \frac{\sqrt{3}}{2}\,r\,\mathrm{d}\mu\,,\\ e^3 &= \frac{\sqrt{3}}{4}\,r\sin\mu(\sin\psi\,\mathrm{d}\theta-\cos\psi\sin\tilde\theta\,\mathrm{d}\tilde\phi)\,,\\ e^4 &= \frac{\sqrt{3}}{4}\,r\sin\mu(\cos\psi\,\mathrm{d}\tilde\theta+\sin\psi\sin\tilde\theta\,\mathrm{d}\tilde\phi)\,, \end{aligned} \tag{A.2}$$

The complex structure of $\mathbb{P}^2$ is given by $z_1 = e_1 + ie_2, z_2 = e_3 + ie_4$. We define the Kähler form and the holomorphic two form as

$$j_0 = \frac{i}{2}(\mathrm{d}z_1\wedge\mathrm{d}\bar z_1 + \mathrm{d}z_2\wedge\mathrm{d}\bar z_2)\,, \tag{A.3}$$

$$\omega_0 = \mathrm{d}z_1\wedge\mathrm{d}z_2\,. \tag{A.4}$$

It is also convenient to define a rescaled $\hat{\omega}_0 = e^{i\tau/2}\omega_0$. A straightforward computation gives the following useful relations:

$$
\begin{aligned}
\mathrm{d}j_0 &= 0\,, \\
\mathrm{d}\,\mathrm{Re}\,\hat{\omega}_0 &= -\frac{\mathrm{d}\tau+\lambda}{2}\wedge \mathrm{Im}\,\hat{\omega}_0\,, \\
\mathrm{d}\,\mathrm{Im}\,\hat{\omega}_0 &= \frac{\mathrm{d}\tau+\lambda}{2}\wedge \mathrm{Re}\,\hat{\omega}_0\,, \\
\mathrm{d}(\mathrm{d}\tau+\lambda) &= 16 j_0\,,
\end{aligned}
\tag{A.5}
$$

where the connection one-form $\lambda = -3\sin^2\mu\,(\mathrm{d}\psi + \cos\tilde{\theta}\,\mathrm{d}\tilde{\phi})$ satisfies $\mathrm{d}\lambda = 16 j_0$. Let us notice that j_0 and λ are SU(3) invariant while $\hat{\omega}_0$ is not.

B Supersymmetry equations and pure spinors

In this paper we will be interested in solutions of type IIA supergravity corresponding to warped products of AdS_4 with an internal compact manifold

$$
\mathrm{d}s^2_{10} = e^{2A}\mathrm{d}s^2_4 + \mathrm{d}s^2_6\,, \tag{B.1}
$$

where A is the warp factor.

The solutions are also characterised by non-trivial values for some of fluxes, respecting the symmetries of AdS_4. The NS H-field has only internal indices and the RR fields split

$$
F^{(10)}_p = \mathrm{vol}_4 \wedge \hat{F}_{p-4} + F_p\,, \tag{B.2}
$$

where vol_4 is the unwarped four-dimensional volume. We can use Hodge duality to express the RR fluxes in terms of the internal components only

$$
\hat{F}_{p-4} = \lambda(*_6 F_{6-p})\,, \tag{B.3}
$$

where λ acts on forms as the reversal of all indices $\lambda(F_p) = (-1)^{\mathrm{Int}[p/2]}F_p$.

Generically, a supersymmetric solution of type II supergravity can be characterised by the form of the spinorial parameters solving the supersymmetry constraints. For backgrounds which are of product type, such parameters factorise accordingly into 4 and 6-dimensional spinors. For type IIA a suitable ansatz is

$$
\begin{aligned}
\epsilon_1 &= \zeta_+ \otimes \eta^1_+ + \zeta_- \otimes \eta^1_-\,, \\
\epsilon_2 &= \zeta_- \otimes \eta^2_+ + \zeta_+ \otimes \eta^2_-\,,
\end{aligned}
\tag{B.4}
$$

where ζ_+ is a four dimensional Weyl spinor and η^i_+, with $i = 1, 2$, are two a priori independent six-dimensional Weyl spinors. The subscripts $\pm$ denote positive and negative chirality spinors, in four and six dimensions.

The spinors η^1 and η^2 define an SU(3) structure on M, each. The intersection of the two will define an SU(2) structure on M and a vector z. We can write

$$
\begin{aligned}
\eta_{1\,+} &= a\eta_+ + b\chi_+\,, \\
\eta_{2\,+} &= x\eta_+ + y\chi_+\,,
\end{aligned}
\tag{B.5}
$$

where $\chi_+ = \frac{1}{2} z \cdot \eta_-$. η_- is the complex conjugate of η_+ and $z\cdot$ denotes the Clifford multiplication by the one-form $z_m \gamma^m$.

If the two spinors are everywhere parallel,

$$\eta^1_+ = a\eta_+ \,, \qquad \eta^2_+ = x\eta_+ \,, \tag{B.6}$$

with a and b complex functions, the two SU(3) structures are identified and the manifold admits an SU(3) structure.

A convenient formalism to study supersymmetric flux backgrounds of this type in type II theories is provided by Generalised Complex Geometry [38, 39]. Given a 6-dimension manifold M, one considers the sum of tangent and cotangent bundles, $TM \oplus T^*M$, and then construct the corresponding spinors. These are Spin(6,6) and have a representation in terms of polyforms on M: positive and negative chirality spinors will correspond to even and odd forms, respectively,

$$\Phi_\pm \in \Lambda^{even/odd}(T^*M)\,. \tag{B.7}$$

As far as supersymmetry is concerned, we will be interested in pure spinors, namely vacua of the Clifford algebra. These can be constructed as tensor products of the supersymmetry parameters η^1 and η^2

$$\Phi_\pm = \eta^1_+ \otimes \eta^{2\,\dagger}_2\,. \tag{B.8}$$

The pair of pure spinors (B.8) are also compatible (they have three common annihilators) and therefore define an SU(3) $\times$ SU(3) structure on $TM \oplus T^*M$. In a way the the two SU(3) can be seen as corresponding to the two SU(3) structures associated to the spinors η^1 and η^2. Then, depending on the relation between them, the explicit form of the pure spinors will change.

For SU(3) structure, the pure spinors have a particularly simple form

$$\begin{aligned} \Phi_+ &= \frac{a\bar{x}}{8}\, e^{-iJ}\,, \\ \Phi_- &= -\frac{iax}{8}\,\Omega\,, \end{aligned} \tag{B.9}$$

where J and Ω are the Kähler form and the holomorphic three-form on the manifold. For the general SU(2) case, the pure spinors read

$$\begin{aligned} \Phi_+ &= \frac{1}{8}\Big[a\bar{x}e^{-ij} + b\bar{y}e^{ij} - i(a\bar{y}\omega + \bar{x}b\bar{\omega})\Big] e^{1/2z\bar{z}}\,, \\ \Phi_- &= \frac{1}{8}\Big[i(by\bar{\omega} - ax\omega) + (bxe^{ij} - aye^{-ij})\Big] z\,, \end{aligned} \tag{B.10}$$

with $|a|^2 + |b|^2 = |x|^2 + |y|^2 = e^A$.

Supersymmetric vacua can be found by solving the ten-dimensional supersymmetry constraints and the Bianchi identities for the NS and RR fluxes. In [40, 41], it was shown that the supersymmetry conditions are equivalent to a set of differential equations for the pure spinors $\Phi_\pm$ on the internal manifold M. In type IIA such conditions read

$$(\mathrm{d} - H\wedge)(e^{A-\phi}\,\mathrm{Re}\,\Phi_-) = 0\,, \tag{B.11}$$

$$(\mathrm{d} - H\wedge)(e^{3A-\varphi}\,\mathrm{Im}\,\Phi_-) = -3e^{2A-\varphi}\mu\,\mathrm{Im}\,\Phi_+ + \frac{e^{4A}}{8} * \lambda(F)\,, \tag{B.12}$$

$$(\mathrm{d} - H\wedge)(e^{2A-\varphi}\Phi_+) = -2\mu e^{A-\varphi}\,\mathrm{Re}\,\Phi_-\,, \tag{B.13}$$

where μ is related to the cosmological constant Λ as $\Lambda = -3|\mu|^2$. Notice that the first equation is actually implied by the last one.

Similarly the Bianchi identities can be given in terms of the internal fluxes only

$$\mathrm{d}H = 0 \qquad\qquad (\mathrm{d} - H\wedge)F = 0\,, \tag{B.14}$$

where F is the sum of all internal RR field strengths

$$F = F_0 + F_2 + F_4 + F_6\,. \tag{B.15}$$

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JHEP09(2009)107

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JHEP09(2009)107